Developing Technical Skills via Scale Model Building

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5 March 2008

Abstract

Incorporating a scale model project provides an especially meaningful way to develop and reinforce a wealth of technical skills at an early stage in the elementary school curriculum.

1 Overview

Introducing a scale model building project provides an excellent opportunity to expose and reinforce a wealth of technical skills especially suited to mathematical and scientific cognition and development. These skills range from basic measurement and arithmetic to the formation of associative networks of relations between various mathematical and scientific concepts that provide foundations for abstract algebra, conservation laws, and quantitative analysis and comprehension. Furthermore, students are exposed to the rudiments of dimensional analysis, analytical use of an object representation, the use of symmetry, approximation error, and error checking and analysis. Extensions offer even more opportunities for skill development and exploration that can be tailored to fit the interest of the instructor and student.

2 Required Skills

In order to attain a modicum of understanding of scale model building some basic mathematical skills are required. Full comprehension of the subtler points requires more skills. However, even modest abilities afford comprehension of the basic concept of scale model building. In this case, the more advanced concepts can be introduced as examples of skills soon to be acquired by the students (say in the next grade of school or next semester of study), or as examples of how basic skills are applied to reveal deeper results.

2.1 Measuring

At a minimum the students should be able to use a ruler to measure items, make the same measurement in two different units, and understand fractional unit measurements.

2.2 Mathematics

To comprehend the basic concept of scale model building, the students need to be able to add, subtract, multiply, and divide natural numbers, form a fraction, multiply a natural number and a fraction, multiply two fractions, add fractions with common denominators.

It is helpful if students know how to simplify fractions, form a simple equation, are able to use and substitute values for named quantities (variables), know the long division algorithm, and know the meaning of decimal numbers.

Along the way, students will see examples of addition, rounding, comparison, and multiplication of decimal numbers, long division of natural numbers to form decimal numbers, transformation of decimal number division into natural number division, and solution of simple equations via multiplication and division.

3 Skills, Techniques, and Principles Exposed and Developed

Many of the benefits of using a project like this to teach technical and scientific skills is that the depth of material covered goes deeper and is more subtle than the development of individual arithmetic functions. Moreover, it is the integration and telescoping of levels of meaning that hold the most promise for both engaging and inspiring students.

3.1 Mathematics

One of the challenges of teaching mathematics is to engage multiple levels of meaning so that they become associated during the abstract rule manipulation that is part of mathematics. Scale model building offers the opportunity to associate operations in the real world with mathematical operations.

Comparing lengths provides a good beginning because it allows two fundamental mathematical operations to take on two different meanings for a basic measurement operation in the real world.

Subtraction is easily associated with the absolute comparison of lengths, resulting in a value characterized as an objective comparison result. Similarly, division is equated with the relative comparison of lengths, yielding a result that itself has two meanings: the number of copies that fit, and/or, how many times bigger/smaller.

This leads directly to one of the main associations of scale model building altogether, a basic geometric concept that is a cornerstone of technical and scientific thinking: multiplication by a factor is the abstraction of changing the size by that factor. This is extrapolated in the very nature of the undertaking into the message that multiplying all the lengths by the same number results in the whole object being scaled by that factor. This result is so fundamental that its application to triangles makes its appearance in Euclid's Elements in Book VI as proposition 19 on similar triangles.

Another challenge of teaching mathematics is to impart the use and utility of equations. Here we have a natural situation that allows the formation of a simple intuitive equation that is easily manipulated and solved to obtain results that are used to make a model. Furthermore, this very example of scale model building serves as a useful application of elementary mathematics in the fields of architecture and chemistry to name but two.

Yet another challenge of teaching mathematics is to impart the realization that there are different, but equivalent ways to represent things, with some representations being more useful than others for certain purposes. Within scale model building we have the opportunity to represent lengths first as fractions that are solutions to the scaling equation, then as decimals that approximate these ratios, and finally again as fractions that represent measurements on a ruler. This leads nicely into another important topic in mathematics and science, approximation error. It is instructive to perform a calculation two ways to get two different answers and then explain about magnification of error. Another aspect of approximation that can be touched upon is function approximation via table lookup, the associated concept of table resolution, and discretization.

One of the the most useful mathematical tools is the use of symmetry to reduce a problem. If the object of the model has symmetry, the organization, measurement, and conversion work of scale modeling can be reduced. A bilateral symmetry will reduce these steps by a factor of 2. Two independent bilateral symmetries will reduce it by a factor of 4. The demonstration of this principle lays the foundations for its use by the student later in science and mathematics where it can lead to profound simplifications and insights.

3.2 Technical and Scientific

The more exposure to and practice with technical and scientific skills the student has, the more skills we impart and reinforce. Scale model building offers the opportunity to expose and practice a surprising number of these, not the least of which is measurement.

Measurement is the ability to analytically quantify amount and is absolutely fundamental to technical and scientific endeavors. One of the most basic and useful measurement skills is the ability to measure lengths with a ruler, and the more practice students have, the more proficient they become. Scale model building offers the opportunity for students to measure lengths in the working photo and then again when the model parts are constructed to size. In addition to representing the model measurements both as decimal numbers and as compound fractions for readings on the ruler, the students associate the measurements from three different spaces: the real world measurements, the photo measurements, and the model measurements.

The separation of measurements into these three spaces is reinforced by measuring each space in different units and begins exposing the students gently to another concept of science and engineering: dimensional analysis (numbers with units). This leads naturally to the topic of conversions, which is yet another meaningful application of multiplication and division that the students need so much to practice and reinforce.

Yet another key scientific principle is that of conservations laws. Scale model building provides a conceptual precursor for conservation laws with the concept of a constant scale factor for an entire object. It does this by first focusing on the concept of a scale factor, then its possible variability via perspective views and skew photography, and finally by specifying conditions for the uniformity of a scale factor. Scale model building then follows up on this by using the constant scale factor to generate an equation that is solved for unknown model lengths. In addition to the enormous importance of this, it also serves as a possible launch-point for a complete unit on 3D perspective drawing that offers its own geometric conceptualizations beneficial for further mathematical, scientific, and technical thought.

Finally, scale model building provides the opportunity to integrate approximation error, data analysis and verification, error checking and correction, and spreadsheet use into a comprehensible and meaningful exercise. By having the students organize their data into a spreadsheet, they can begin a step into computer use and programming by automating the conversion of the measurements from the photo space into the model space. (A possible programming extension is to program the table lookup for the conversion back into symbolic fractions for use with a ruler.) If the object of interest provides enough structural regularity and detail (think of a bridge with many vertical supports of varying size), once the measurements are entered they can be graphed to perform a visual check for errors. If the student is lucky, there will be at least one error which will immediately obvious and can then be corrected. Either way, the visual verification impresses the utility of data visualization as a technical tool.

4 Cognitive, Higher Level Benefits

Over and beyond the direct presentation and practice of the skills and techniques already mentioned, incorporation of a scale model building project has the following additional higher level cognitive benefits. Even though some of these benefits have been mentioned above, they are so important to later technical thinking ability that they are mentioned here again in this specific context.

4.1 Connecting Physical Reality to Mathematical Operations and Numbers

A key component of mathematical and scientific thinking involves abstracting common forms into a single pattern. One of the challenges is to enable students to work with the abstractions while simultaneously activating concrete representations of the objects involved that give an intuition and innate understanding of the abstractions that are being manipulated. In this context, the utility of a scale model building project that equates multiplication by numbers with changing the size of an object becomes obvious. Further thought illuminates the fact that the earlier you can make this association, the better for helping a young mind organize the mathematical and physical world around them. It is even possible to imagine that, once the student has attained this association and uses it successfully, it imparts an enjoyment of mathematical learning, a sense of accomplishment, and appreciation for the abstraction mechanism that our brains employ.

And yet, this is only the primary association. Along the way we can form the associations between subtraction and absolute length comparisons, division and relative length comparisons, division and fractions, and fractions and decimals. These associations form the components that later allow the student to build, fit, and use abstract patterns to represent and analyze relations in a mathematical and scientific way.

As important as each of these concepts and associations is individually, they are even more important to a young mind when they are integrated and interrelated into a single comprehensive use.

4.2 Exploration and Reinforcement of the Interrelations Between Division, Fractions, Decimals, Multiplication, and Scaling

Approaching the topic of scaling through division as a relative comparison of lengths provides a number of useful associations for the student. First there is the association between dividing and forming a fraction. As attention focuses on the result of the division, it is interpreted as the relative comparison of lengths. This gives the association between the value of the fraction and a scale factor. Furthermore, there is the association between multiplication by the fraction, multiplication by the scale factor, and the formation of the resulting scaled length which is determined through the mechanism of multiplication and division. This leads naturally into and explanation of cancellation, providing further associations of multiplication, division, and fractions. Finally, there is the association of fractions and decimal approximations that result from carrying out long division in order to get and approximation to the resulting scaled length. This also offers the opportunity for an extension the explores repeating decimal patters for fractions.

The relations and associations between arithmetical division, fractions, multiplication by fractions, cancellation, long division, and decimal numbers all serve to form and solidify a conceptual network that gives meaning to the concept of division. Moreover, examples of multiplication of a fraction by the value of the denominator to yield cancellation exposes the student in a rudimentary way to the concept that multiplication and division are inverse operations.

4.3 Analytical use of a Representation of an Object

One of the subtler higher level skills developed by scale model building via photograph is the analytical use of a representation of an object. The photograph captures the requisite spatial relationships into a representation that suffices for the task at hand. The consideration and discussion of properties the representation must possess to make it usable (3D perspective considerations) form a fundamental step toward the ability to develop abstract representations: the distillation of selected characteristics into a core representation that can be analyzed and manipulated symbolically. The use of the representation (measuring the photograph) provides feedback to reinforce and integrate representation property considerations. Even though the use of a photograph to represent an object is only one step removed from the object itself, it is a step. The extrapolation of a sequence of such steps leads to the ability form abstractions that result in abstract algebras, equivalences of space and time, and prepare the student for further work in mathematics, physics, computer science and more.

4.4 Use of Elementary Mathematics to Perform an Otherwise Daunting Task

When the object being modeled is complicated with a multitude of non-integral ratios of lengths, measurements and mathematics provide the keys to breaking down the task into understandable pieces for the young mind. Even without the full complement of mathematical skills, comprehension of the basic principle of scale model building hinges only on the mathematical concepts of fractions and multiplication by fractions. Using these elementary mathematical skills to understand how a scale model can be built not only prepares the student to use mathematics and quantitative methods to comprehend the world around them but also validates the teaching of elementary mathematics.

4.5 Use of Elementary Mathematics to Solve a Real-World Problems

Over and above the educational preparation and comprehension that elementary mathematics brings is the result that it enables, the building of the scale model. This in itself is useful in the fields of aeronautical design, architecture, chemistry, and more. Solution of real-world problems is often the most potent motivation for learning mathematics, science, and technical skills. This again prepares the student to use mathematics and quantitative methods to deal with the world around them and validates the teaching of elementary mathematics.

4.6 Multimodal Integration of Skills

The Multimodal aspect of scale model building helps engage the student and build a stronger, more fundamental association between all components of the project. The more senses that are activated, the more associational links can be formed between the internal representations in a young mind. More specifically, as the student views the object to be modeled, a visual representation is formed in the visual cortex. When the photo is viewed, the visual representation is reactivated. Similarly, as the student handles parts of the model, spatial and visual representations are simultaneously active. As the student measures the pieces being constructed, the mathematics that gave the lengths is also activated to the extent of the associational networks that have been formed. As the student intends to construct, or manipulate the model pieces, the prefrontal cortex forms and strengthens links with the various representations and the motor cortex. Finally, as the model takes shape, the spatial representation, visual representation, measurement activities, and to some extent the mathematics that enabled it all are activated, strengthened, and grouped together as a named concept of scale modeling or some equivalent.

5 Curriculum Options

Several different plans for integrating the project into the elementary school curriculum come to mind. Of the options listed below, incorporating a scale model building project as a third grade enrichment extension seems to be the most demanding for the instructor as this requires the most individual interactions. Assuming only modest interest in the project, the special subject extraction or after school enrichment class is probably the least demanding. As a word of warning: I only have experience with the third grade enrichment extension! The remaining curriculum options are only speculations of my imagination.

5.1 3rd Grade Enrichment Extensions

One possible way to integrate the project into the third grade as a enrichment extension actually begins between second and third grades. The long break over the summer offers the time required to cover many of the topics required.

This track starts with an early understanding and mastery of multiplication, moves on to division, and then introduces and develops an understanding of and the arithmetic of fractions, including simplification and canceling. Once this is accomplished, the student can continue with the introduction of algebra by working through appropriate sections of a book such as Lynette Long's *Painless Algebra*. This also gives a chance to practice the multiplication, division, and fraction skills. Throughout the third grade school year, the student can acquire multidigit multiplication, long division, an understanding of the decimal system, and how long division yields decimal numbers. At this point the student is prepared for the application of these skills to scale model building.

5.2 4th Grade in Class Curriculum

This method involves reviewing and introducing the required mathematics as part of the class, with the scale model building project occurring soon after the prerequisite mathematics. The project could proceed with the selection of objects to model, presentation of the scale modeling theory together with exercises, and finish with the building of the models. Of course, there are options for integrating the project into the class and extending the project to overlap with other fourth grade educational requirements.

5.3 Special Subject Extraction Class or After School Enrichment Class

Much as for the fourth grade class method, either of these options involves gathering students together with an instructor to cover the material as described above. The main differences here are that: 1) The class is gathered for sole purpose of scale model building. 2) It can be limited in size, via prerequisites or other constraints. 3) The division between review and new material, and selection of material to to fit the class can be more easily directed by the instructor.

6 Extensions

Scale model building provides a natural launch point for the following worthwhile extensions. Rather than delving into them during scale model building, use them for further study after the completion of the model. A cohesive, focused approach to scale model building will shorten the project, avoid extraneous complications, and strengthen the association and comprehension of the concepts involved.

In the case of leading a class of diverse students where increasing the tempo is problematic, use them as individual enrichment extensions for students that assimilate the material quickly and/or are more productive when challenged.

6.1 3D Perspective Drawing

Only the rudiments of 3D perspective are touched on during scale-model building, but an in-depth exploration and development of perspective drawing skills would also serve to develop valuable technical skills.

Ernest R. Norling's *Perspective Made Easy* is a simple, straightforward introduction to 3D perspective.

6.2 Repeating Decimal Patterns for Fractions

Some students will show an affinity for performing the long division algorithm. If this is the case, and the opportunity to explore this idea presents itself, further study affords a deeper understanding of rational numbers and and irrational numbers. The key to the investigation is that once all of the non-zero digits of the dividend are exhausted by the algorithm and only zeros are available for carrying down to the remainder, a repeating decimal pattern will emerge. This also makes for a simple presentation of a comprehensible mathematical proof, chaining together the reasoning about: the limited choices for the remainder, repetition of a remainder value, the division algorithm operation in such a case, and the sequence of resulting quotient digits.

6.3 Automating Conversion of Decimal Numbers into Compound Fractions via Table Lookup

For students that show an affinity for spreadsheets and automating calculations, developing macros and formulas in a spreadsheet is a useful skill in and of itself and can be a step toward more general programming skills later. Automating the conversion of a decimal number back into an approximating fraction via a lookup table is one extension along these lines. One way to proceed with such an extension could start by forming the lookup table of fractions (useful even if the extension stops here), separating a number into integral and fractional components, finding the closest entry in the table, and recombine the number using the integral part and the index.

6.4 Investigate and Report on the Use of Scale Models

There are many fields that employ scale models, such as: aeronautical design, architecture, automotive design, chemistry, mechanical design, movie production, nautical design. One possible extension for scale model building is for the student to investigate and report as many different fields as can be found that make use of scale models. Another possible extension is for the student to investigate and report the details of how and why scale models are used in one specific field.

6.5 Report on How New Use of Scale Model Helps

One of the most creative and rewarding extensions may be for the student to come up with some new way that a scale model may solve some problem or provide some new understanding.

7 Summary

The introduction of scale model building into the elementary curriculum provides such an overwhelming opportunity to develop a wealth of mathematical, scientific and technical skills, that it is hard to appreciate the magnitude of the step toward analytical thinking that this affords young minds. Over and above the practice of individual technical skills, it forms and integrates an associative network of rudimentary mathematical and scientific concepts that allows the student to understand and manipulate the world in a quantitative way. It provides a solution to a interesting, enjoyable, and meaningful real-world problem and validates the teaching of mathematics to the student (and others).

References

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